Name________________________________ Date__________

6.6 Enrichment and Extension

Exploring Newton's Law of Cooling

The natural base e is used to solve the equation that models Newton's Law of Cooling. The equation states that the difference in temperature between an object and its surroundings decreases exponentially as a function of time.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

where $T(t)$ = the temperature after t minutes, T_0 = the initial temperature, T_s = the surrounding temperature, t represents time, and $-k$ represents a constant rate of decrease in the temperature difference.

Example: When the constant surrounding temperature is 65°F, an object will cool from 160°F to 120°F in 30 minutes. After how long will the object's temperature be 70°F?

Solution:

$120 = 65 + (160 - 65)e^{-k(30)}$ Substitute the given values into the function.

$\frac{55}{95} = e^{-30k}$ is the same as $\frac{95}{55} = e^{30k}$ Simplify to solve for k.

$\ln\left(\frac{95}{55}\right) = \ln(e)^{30k} \rightarrow 0.5465 = 30k \rightarrow k = 0.0182$

Use k to determine when the temperature is 70°F.

$$70 = 65 + (160 - 65)e^{-0.0182t}$$

$$\ln\left(\frac{5}{95}\right) = -0.0182t$$

$$t \approx 2 \text{ hours and 42 minutes}$$

In Exercises 1 and 2, use $T(t) = T_s + (T_0 - T_s)e^{-kt}$ to solve the problem.

1. At a local restaurant, the cook prepares enough soup at night so that there is plenty of soup for customers the next day. Refrigeration is necessary, but the soup is too hot at 220°F to put directly into the fridge. It needs to be no more than 70°F. The cook cooled the soup by placing it in a sink of water constantly running at 40°F. After 10 minutes, the soup had cooled down to 140°F. After how long can the cook place the soup in the fridge?

2. Milk is taken out of the fridge and left on a table. Its temperature is 35°F and the house is a constant 70°F. The temperature of the milk rose 5°F after 1 hour. However, milk is unsafe to consume at temperatures over 45°F. How long can the milk be left out and still be safe to consume?

Name ____ Date _________

6.6 Extra Practice

In Exercises 1–6, solve the equation.

1. $9^{3x-5} = 81^{3x+2}$ **2.** $7^{x} = 32$ **3.** $9^{3x+6} = \left(\frac{1}{3}\right)^{8-x}$

4. $6^{4x} = 13$ **5.** $2e^{3x} + 6 = 10$ **6.** $4e^{2x} - 7 = 1$

7. Fifty grams of radium are stored in a container. The amount R (in grams) of radium present after t years can be modeled by $R = 50e^{-0.00043t}$.

a. After how many years will only 20 grams of radium be present?

b. Seventy-five grams of radium are stored in a different container. The amount R (in grams) of radium present after t years can be modeled by $R = 75e^{-0.00043t}$. Will it take *more years* or *fewer years* for only 20 grams of the radium in this container to be present, compared to the answer in part (a)? Explain.

In Exercises 8–13, solve the equation.

8. $\ln(5x - 2) = \ln(x + 6)$ **9.** $\log(3x + 5) = \log 6$

10. $\log_2(3x + 12) = 4$ **11.** $\log_3(3x + 7) = \log_3(10x)$

12. $\log_2(x^2 - 2x + 1) = 4$ **13.** $\log_3(x^2 + x + 7) = 3$

In Exercises 14–17, solve the equation. Check your solution(s).

14. $\ln x + \ln(x - 2) = 5$ **15.** $\log_5 2x^2 + \log_5 8 = 2$

16. $\log_3(-x) + \log_3(x + 8) = 2$ **17.** $\log_2(x + 2) + \log_2(x + 5) = 4$

In Exercises 18–20, solve the inequality.

18. $e^{x-2} < 8$ **19.** $\ln x > 5$ **20.** $-2 \log_3 x + 2 \leq 10$

21. You deposit \$2000 in Account A, which pays 2.25% annual interest compounded monthly. You deposit another \$2000 in Account B, which pays 3% annual interest compounded monthly. When is the sum of the balance in both accounts at least \$5000?